FEMSSISA MATHEMATICS OLYMPIAD: Grade 8 - 12

E-mail : Maths@saolympiads .co.za website: www.saolympiads .co.za

MATHEMATICS OLYMPIAD GRADE TWELVE

DATE: 5 AUGUST 2024 – 8 AUGUST 2024

TIME: 90 MINUTES

INSTRUCTIONS:

1. This booklet has 15 multiple choice and 5 open ended questions.
2. Use the answer sheet provided.

   Circle the letter corresponding to your answer.
3. All working details must be done in the space provided.
4. Calculators are not permitted.
5. Diagrams are not necessarily drawn to scale.
6. The first 15 problems carry 1 mark each and the next 5 carry 2 marks each.
7. You have 90 minutes for the paper which works out to an average of 4.5 minutes per question.
8. Read the questions carefully before answering.
9. Visit the website: www.saolympiads.co.za.

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Grade Twelve Mathematics Olympiad

1. Which term of this arithmetic sequences is equal to 145?
   5; 12; 19; 26; ….
   (A) 19th (B) 21st (C) 23rd (D) 25th

2. Write down the period of f, if f = $\{(x;y): y = \cos 2x; x \in [-360^0; 360^0]\}$
   (A) 360^0 (B) 270^0 (C) 180^0 (D) 135^0

3. Determine the values of k for which the equation has 2 unequal real roots:
   x = $4 \pm \sqrt{k^2 - 2k - 3}$
   (A) $k - 1$ (B) $k > 3$ (C) $-1 < k < 3$ (D) $k < -1 \ or \ k > 3$

4. Write down the amplitude of g if g = $\{(x;y): y = -2\sin x + 1; x \in [-360^0; 360^0]\}$
   (A) 0 (B) 1 (C) 1.5 (D) 2

5. Rewrite h given by y = - $2x^2$ + 5x - 3 in the form:
   y = $a(x-p)^2 + q$
   (A) y = $-2(x - \frac{5}{4})^2 - \frac{1}{8}$ (B) y = $-2(x + \frac{5}{4})^2 - \frac{1}{8}$
   (C) y = $-2(x - \frac{5}{4})^2 + \frac{1}{8}$ (D) y = $-2(x + \frac{5}{4})^2 + \frac{1}{8}$

6. Solve for x:
   $4x^3 - 19x - 15 = 0$
   (A) $x = \frac{5}{2}; 1; \frac{-3}{2}$ (B) $x = \frac{-5}{2}; 1; \frac{-3}{2}$ (C) $x = \frac{5}{2}; -1; \frac{-3}{2}$ (D) $x = \frac{5}{2}; -1; \frac{3}{2}$

7. The distance ‘s’ metres travelled by an insect in ‘t’ minutes is given by $s(t) = 7t - t^2$.
   Find the average speed of the insect between t = 1 and t = 3.
   (A) 2 m/min (B) 3 m/min (C) 3.5 m/min (D) 4 m/min

8. Determine: $\frac{dy}{dx}$ if $xy = 3y + x^2 + 2x - 15$
   (A) x (B) 1 (C) 2 (D) 3x

9. Solve for ‘x’ :
   $\sqrt{x+3} - \frac{8}{\sqrt{x+3}} = 2$
   (A) 1 (B) -1 (C) -13 (D) 13

10. If $\sin\theta = -\frac{4}{5}$ and $\theta \in [0^o;270^o]$.

Then calculate:

$$\frac{1}{\sin(\theta-90^o)}$$

(A) $\frac{5}{3}$ (B)) $\frac{5}{4}$ (C)) $\frac{-5}{3}$ (D)) $\frac{5}{-4}$

11. Determine the coordinates of the point of inflection of:

$y = x^3 - 3x^2 - 10x + 24$

(A) (1;12) (B) (2;0) (C) (3;-8) (D) (0;24)

12. Solve for x:

$3 \leq \frac{4}{2-x}$

(A) $\frac{2}{3} \leq x \leq 2$ (B) $\frac{2}{3} \leq x < 2$ (C) $\frac{2}{3} < x < 2$ (D) $-2 < x \leq \frac{2}{3}$

13. Write as a single ratio:

$\sqrt{3}\sin(x+30^o) + \sin(x-60^o)$

(A) -2 sin x (B) 2 cos x (C) sin x (D) 2 sin x

14. A small factory can produce 400 pairs of shoes per week and sell them at R320 per pair. For each increase of R10 per pair one less pair is sold. At what price must each pair be sold to maximize receipts?

(A) R1760 (B) R1960 (C) R2160 (D) R2360

15. The scores with common difference were recorded as follows:-

30; 39; 48; 57; ……; 201; 210; 219; 228.

Determine the mean of these scores.

(A) 120 (B) 129 (C) 138 (D) 147

16. If f : x $\longrightarrow \frac{3x-1}{x+2}$ then determine the coordinates of the intersection of the asymptotes.

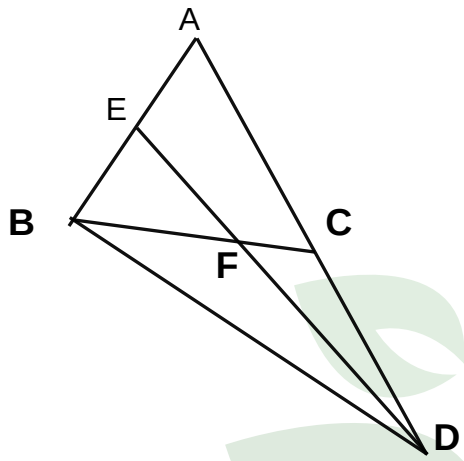
17. If $4^x = 24^k$ and $6^y = 24^k$ then write down k in terms of x and y.

18. Two friends participated in the City to City Marathon. The probability that A would complete the marathon is 0.6. The probability that B will complete the marathon is 0.8.
What is the probability that at least one will complete the marathon**?**

19. What is the maximum value of E if p is a real number?

$$E = \frac{48}{p^2 - 2p + 3}$$

20. ABC is a triangle. AC is produced to D such that C is the midpoint of AD. E is a point on AB such that BE is $\frac{2}{3}$ the length of AE. DE intersects BC at F.
What is the ratio of BF: BC?



MARKS: 1-15: 15 X 1 = 15
16-20: 5 X 2 = 10

TOTAL = 25